Student Workbook

Matrix Operations and Applications

Name: ________________________________ Date: _________________ Class: _________

ESSENTIAL QUESTION

How does matrix multiplication encode a system of linear equations, and why is the order of multiplication critical — what does it mean that matrix multiplication is not commutative?

Lesson Overview

A matrix is a rectangular array of numbers with m rows and n columns (an m×n matrix). Entry a_{ij} denotes the value in row i, column j. Two matrices are equal only if they have the same dimensions and all corresponding entries match.

Add / Subtract → Scale → Multiply (Row × Column)

Add/Sub	Add or subtract corresponding entries — matrices must have the same dimensions.
Scalar ×	Multiply every entry by the scalar k.
Mat. Mult.	A(m×n)·B(n×p)=C(m×p). Entry c_{ij} = (row i of A)·(column j of B). Columns of A must equal rows of B.
Not Comm.	AB ≠ BA in general — the order of multiplication matters.
Identity	I_n is the n×n matrix with 1s on the diagonal, 0s elsewhere; AI = IA = A.

Worked Examples

EXAMPLE 1

Given A = [[1,2],[3,4]] and B = [[5,6],[7,8]], find A+B, A−B, and 3A.

- A+B: add corresponding entries → [[1+5,2+6],[3+7,4+8]] = [[6,8],[10,12]]
- A−B: subtract corresponding entries → [[1−5,2−6],[3−7,4−8]] = [[−4,−4],[−4,−4]]
- 3A: multiply every entry by 3 → [[3,6],[9,12]]

Answer: A+B = [[6,8],[10,12]], A−B = [[−4,−4],[−4,−4]], 3A = [[3,6],[9,12]]

EXAMPLE 2

Find AB where A = [[1,2,3],[4,5,6]] (2×3) and B = [[7,8],[9,10],[11,12]] (3×2).

- A is 2×3, B is 3×2 → result is 2×2
- c_{11} = 1·7+2·9+3·11 = 58. c_{12} = 1·8+2·10+3·12 = 64
- c_{21} = 4·7+5·9+6·11 = 139. c_{22} = 4·8+5·10+6·12 = 154

Answer: AB = [[58, 64],[139, 154]]

EXAMPLE 3

Show AB ≠ BA for A = [[1,2],[3,4]] and B = [[0,1],[1,0]].

- AB: c_{11}=1·0+2·1=2; c_{12}=1·1+2·0=1; c_{21}=3·0+4·1=4; c_{22}=3·1+4·0=3 → AB=[[2,1],[4,3]]
- BA: c_{11}=0·1+1·3=3; c_{12}=0·2+1·4=4; c_{21}=1·1+0·3=1; c_{22}=1·2+0·4=2 → BA=[[3,4],[1,2]]
- [[2,1],[4,3]] ≠ [[3,4],[1,2]]

Answer: AB = [[2,1],[4,3]], BA = [[3,4],[1,2]], AB ≠ BA

EXAMPLE 4

Write the system 2x + 3y = 7, x − 4y = −2 as a matrix equation Ax = b.

- Coefficient matrix A: row 1 = [2,3], row 2 = [1,−4]
- Variable vector x = [[x],[y]], constant vector b = [[7],[−2]]

Answer: [[2,3],[1,−4]] · [[x],[y]] = [[7],[−2]]

EXAMPLE 5

Find x and y if [[2x+1, 3],[y−2, 5]] = [[7, 3],[4, 5]].

- Set corresponding entries equal: 2x+1 = 7 and y−2 = 4
- 2x+1 = 7 → x = 3. y−2 = 4 → y = 6

Answer: x = 3, y = 6

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Given A = [[2,−1],[0,3]] and B = [[1,4],[−2,1]], find A+B, A−B, and 2A−3B.

Hint: Perform each operation entry by entry.

2 Find AB where A = [[1,0],[2,1]] and B = [[3,1],[−1,2]].

Hint: Use the row-times-column rule for each entry.

3 Find BA for the same A and B as in problem 2. Is AB = BA?

Hint: Compare your answers from problems 2 and 3.

4 Write the system $3x - 2y + z = 5$, $x + y - z = 2$, $2x + 3y + z = 8$ as a matrix equation $Ax = b$.

Hint: The coefficient matrix A has the coefficients of x, y, z in each row.

5 Find x, y, z if [[x+y, 2z],[3, x−z]] = [[5, 4],[3, 1]].

Hint: Set corresponding entries equal and solve the resulting system.

Key Vocabulary

Matrix	A rectangular array of numbers with m rows and n columns.
Entry a_{ij}	The element in row i, column j of matrix A.
Dimensions	The size of a matrix, written as m×n (rows × columns).
Scalar Mult.	Multiplying every entry of a matrix by a constant. Ex: 3·[[1,2],[3,4]] = [[3,6],[9,12]].
Mat. Mult.	AB where entry c_{ij} = (row i of A)·(column j of B); requires columns of A = rows of B.
Identity I_n	The n×n matrix with 1s on the main diagonal and 0s elsewhere; AI = IA = A.

Quick Check Quiz

Circle the letter of the correct answer for each question.

1 **A 3×4 matrix multiplied by a 4×2 matrix gives a matrix of size:** Multiple Choice

A 3×2
B 4×4
C 3×4
D 4×2

2 **Matrix multiplication is:** Multiple Choice

A Always commutative
B Never commutative
C Commutative only for square matrices
D Not commutative in general

3 **The identity matrix I_2 is:** Multiple Choice

A [[0,0],[0,0]]
B [[1,0],[0,1]]
C [[1,1],[1,1]]
D [[2,0],[0,2]]

4 **If A is 2×3 and B is 4×3, then AB is:** Multiple Choice

A 2×4
B 2×3
C Undefined
D 3×3

5 **2 · [[1,3],[−2,0]] =** Multiple Choice

A [[2,6],[−4,0]]
B [[1,3],[−2,0]]
C [[2,3],[−2,0]]
D [[3,5],[−4,2]]

Common Mistakes to Avoid

✗ Multiplying matrices entry-by-entry (like addition).

✓ Matrix multiplication uses the row-times-column dot product. Entry c_{ij} = (row i of A)·(column j of B).

✗ Assuming AB = BA.

✓ Matrix multiplication is NOT commutative. Always check — AB and BA may be different matrices (or one may not even be defined).

✗ Multiplying matrices with incompatible dimensions.

✓ For AB to be defined, the number of columns of A must equal the number of rows of B. Check dimensions first.

✗ Confusing the order of rows and columns in the dimension notation.

✓ An m×n matrix has m ROWS and n COLUMNS. The entry a_{ij} is in row i, column j.

Math Tips

★ Dimension check first: $(m\times n)(n\times p) = m\times p$. The inner dimensions must match; the result has the outer dimensions.

★ The identity matrix is the matrix equivalent of the number 1: $AI = IA = A$ for any compatible matrix A.

★ To encode a system $Ax = b$: A is the coefficient matrix, x is the column vector of variables, b is the column vector of constants.

★ Matrix addition is commutative ($A+B = B+A$) and associative. Matrix multiplication is associative but NOT commutative.

★ When computing AB, think of it as: each column of AB is A times the corresponding column of B.